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Detecting Faults from Encoded Information

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Abstract—The problem of fault detection for linear continuous-time systems via encoded information is considered. The encoded information is received at a remote location by the monitoring device and assessed to infer the occurrence of the fault. A class of faults is considered which allows to use a simple decision logic as monitoring device.

I. INTRODUCTION

In complex systems information is exchanged through communication channels. Since information can travel in the form of bits, devices (*encoders*) are needed to convert the information (which can come in the form of an analog quantity) into a binary form. The information can be used for a variety of purposes. In the control community, many have focused their attention on the problem of estimation and control (see e.g. [4], [7], [19], [20], [16], [3], [13], [5], [15], [9], [2], [17], [18], [8], [6], [1], [10], [12] and references therein). Among these contributions, we single out papers such as [3], [17], [18] where it was pointed out the potentiality of *time-varying* encoders. In this paper, the encoded information is used to a different purpose: the purpose of detecting faults. We consider the scenario in which a stream of bits is originates from a process under monitoring and we aim to design a device which, upon reception of the information from the channel, assess it and decide whether or not a fault is occurring.

In this paper the process under consideration has the following form

$$\dot{x}(t) = Ax(t) + Bu(t) + Mm(t), \quad t \geq 0, \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state of the process, $u(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}^m$ is a vector-valued and measured input signal and $m(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a fault signal. Process (1) can be interpreted as a model of one of many sub-components a a complex system is comprised of. By fault it is meant a signal which is identically zero over the interval $[0, \bar{t})$ and which becomes non zero for the first time at \bar{t} . The time behavior of $m(\cdot)$ is otherwise unknown. The (fault) vector M is nonzero to avoid triviality.

If measurements are available in the form

$$y = \Gamma x,$$

with $\Gamma \in \mathbb{R}^{p \times n}$ a matrix for which the pair (Γ, A) is observable, and no encoding is present, the problem of detecting faults is a trivial one. As a matter of fact, any Luenberger

observer for (1) with internal state ξ and diagnostic signal r given by $y - \Gamma\xi$ allows to detect any fault. In other words, the diagnostic signal will be (practically) zero before the occurrence of the fault and will become nonzero as soon as a fault occurs. If encoding is present then this is not possible any longer, and examples are easily found to show that there are classes of faults which can not be detected at all. In this paper we address the problem by considering a class of faults for which the detection problem becomes fairly tractable. These faults in particular lead to an elementary decision logic as detection unit. The time-varying encoder for continuous-time linear systems employed in this paper merges features from both [11] and [17], [18]. In particular, we borrow from [11] the idea to encode the information coming from the process only at discrete times and to reconstruct the inter-sample behavior through the encoded samples and the mathematical model of the process. On the other hand, in [17] the authors consider a linear discrete-time system and exploit the Jordan form of the dynamic matrix to decrease the number of bits used to encode the information. To extend their results to linear continuous-time systems ([18]), the authors assume piece-wise constant inputs $u(\cdot)$, consider the corresponding sampled-data system and apply the methods found for discrete-time systems. In doing this, they are forced to restrict the set of values which the sampling time T can take on. The time-varying encoder considered in this paper do not use the sampled-data version of system (1) and therefore do not put any restriction on the sampling time T . On the other hand, as in [17], [18], it exploits the Jordan form of the dynamic matrix to reduce the number of bits used to encode information (compare with [11]).

The preliminaries needed to state the results in a concise manner are introduced in Section II. The encoder is found in Section III. The formulation of the problem and a solution are proposed in Section IV. Conclusions are found in Section V.

II. PRELIMINARIES

Let $\Phi \in \mathbb{R}^n \times \mathbb{R}^n$ be a nonsingular matrix for which

$$F = \Phi A \Phi^{-1}$$

has the following special structure:

$$F = \text{block.diag}(F_1, \dots, F_p, F_{p+1}, \dots, F_q), \quad (2)$$

$$F_j \in \mathbb{R}^{n_j \times n_j}, \sum_{j=1}^q n_j = n$$

where, for $j = 1, \dots, p$, the F_j 's are Jordan blocks associated to real eigenvalues $\lambda_j \in \mathbb{R}$, i.e.

$$F_j = \begin{pmatrix} \lambda_j & 1 & \dots & 0 \\ 0 & \lambda_j & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_j \end{pmatrix}, \quad (3)$$

whereas for $j = p+1, \dots, q$ the F_j 's are Jordan blocks associated to complex eigenvalues $a_j \pm i\omega_j \in \mathbb{C}$, i.e.

$$F_j = \begin{pmatrix} \rho_j & I_2 & \dots & 0 \\ 0 & \rho_j & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \rho_j \end{pmatrix}, \quad (4)$$

with

$$\rho_j = \begin{pmatrix} a_j & -\omega_j \\ \omega_j & a_j \end{pmatrix}, \quad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Associated to F we also define the matrices \bar{F} and S . Matrix \bar{F} is defined as

$$\bar{F} = \text{block.diag}(\bar{F}_1, \dots, \bar{F}_p, \bar{F}_{p+1}, \dots, \bar{F}_q),$$

where, in correspondence to real eigenvalues λ_j , we have

$$\bar{F}_j = \begin{pmatrix} e^{\lambda_j T} & T e^{\lambda_j T} & \dots & \frac{T^{n_j-1}}{(n_j-1)!} e^{\lambda_j T} \\ 0 & e^{\lambda_j T} & \dots & \frac{T^{n_j-2}}{(n_j-2)!} e^{\lambda_j T} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{\lambda_j T} \end{pmatrix}.$$

On the other hand, in correspondence to complex eigenvalues $a_j \pm i\omega_j$ we have

$$\bar{F}_j = \begin{pmatrix} E_j & T E_j & \dots & \frac{T^{n_j-1}}{(n_j-1)!} E_j \\ 0 & E_j & \dots & \frac{T^{n_j-2}}{(n_j-2)!} E_j \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & E_j \end{pmatrix},$$

where

$$E_j := e^{a_j T} r(\omega_j T) := e^{a_j T} \begin{pmatrix} \cos(\omega_j T) & -\sin(\omega_j T) \\ \sin(\omega_j T) & \cos(\omega_j T) \end{pmatrix}.$$

Matrix S is defined as

$$S = \text{block.diag}(S_1, \dots, S_p, S_{p+1}, \dots, S_q),$$

with

$$S_j = \begin{pmatrix} 1/2^{R_j} & 0 & \dots & 0 \\ 0 & 1/2^{R_j} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1/2^{R_j} \end{pmatrix} \in \mathbb{R}^{n_j \times n_j}.$$

For $i = 1, \dots, p$, the integers R_j are required to satisfy the inequality

$$R_j > \max\{0, \log_2 e^{\lambda_j T}\}, \quad (5)$$

whereas for $j = p+1, \dots, q$, the integers R_j are required to satisfy the inequality

$$R_j > \max\{0, \log_2 e^{a_j T}\}. \quad (6)$$

We set

$$R := \sum_{j=1}^q n_j R_j.$$

The number of bits used to encode information will be equal to 2^R . Finally, we define H as the matrix

$$H = \text{block.diag}(H_1, \dots, H_p, H_{p+1}, \dots, H_q),$$

where $H_j = I_{n_j}$, for $j = 1, \dots, p$, and

$$H_j = \text{block.diag}(r^{-1}(\omega_j T), \dots, r^{-1}(\omega_j T)) \in \mathbb{R}^{n_j \times n_j},$$

for $i = p+1, \dots, q$. We note the following result, which is the analogous of Lemma 4.1 in [17]:

Lemma 1: For each $k \geq 0$, $H^{-k} \bar{F} H^k = \bar{F}$.

Denote by \mathcal{R}_C^L the hyper-rectangle with $C \in \mathbb{R}^n$ as the center and whose edges have lengths given by the entries of $L \in \mathbb{R}_+^n$. The following holds by Lemma 4.2 in [17]:

Lemma 2: Let k be any non-negative integer and (x, \bar{x}) any pair in $\mathbb{R}^n \times \mathbb{R}^n$. If $H^k \Phi(x - \bar{x})$ belongs to \mathcal{R}_0^L , for some vector $L \in \mathbb{R}_+^n$, then a vector $\hat{x} \in \mathbb{R}^n$ can be determined for which $H^k \Phi(x - \hat{x})$ belongs to \mathcal{R}_0^{SL} .

Remark. Note that $H^k \Phi(x - \bar{x}) \in \mathcal{R}_0^L$ implies $H^k \Phi x \in \mathcal{R}_{H^k \Phi \bar{x}}^L$. For $i = 1, 2, \dots, n$, divide edge i of $\mathcal{R}_{H^k \Phi \bar{x}}^L$ into 2^{R_i} parts. This will result into a partition of $\mathcal{R}_{H^k \Phi \bar{x}}^L$ into 2^R subregions. Among these subregions single out the one to which $H^k \Phi x$ belongs. Denote by \tilde{x} the centroid of this subregion. By construction, for each $i = 1, 2, \dots, n$,

$$|(H^k \Phi x)_i - \tilde{x}_i| \leq L_i / 2^{R_i+1},$$

where the index j is the smallest positive integer in the set $\{1, 2, \dots, p\}$ for which $\sum_{\ell=1}^j n_\ell \geq i$. Vector \hat{x} is then obtained by setting

$$\hat{x} := \Phi^{-1} H^{-k} \tilde{x}. \quad \triangleleft$$

III. ENCODERS

In this section we introduce the device which encode the available information. Were we free to design an encoder only for monitoring purposes, the problem would be easily solvable using a single bit (see [14]). However, we are interested in detecting faults starting from packet of bits which have not been necessarily encoded for fault detection. In other words, we shall consider encoders which generate packets of bits which can serve to state estimation or feedback stabilization as well, and not only for process monitoring. One can envision the situation where the packet received by a control unit is used to devise a control action whereas the same packet received by a monitoring unit is used to possibly generate an alarm signal. To keep small the complexity of the encoders and the number of bits used to encode information, we shall neither modify the structure of the encoder nor use an extra bit to signal occurrence of a fault, but we shall rather design the monitoring unit to extract information on the status of the process from the packets of bits which the unit is receiving from the channel.

The encoder works under the following assumptions:

Assumption 1: Full-state information is available, i.e.

$$C = I_n \triangleleft$$

Remark. Although this assumption can be easily relaxed, in this paper, for the sake of simplicity, we choose to examine the only case of full-state measurements. \triangleleft

Assumption 2: The input $u(\cdot)$ is available to the encoder.

\triangleleft

Loosely speaking, the functioning of the encoder is as follows. Every T units of time, it acquires the state sample $x(kT)$, where k is a non-negative integer. At the same time, the encoder builds a compact subset of the state space (the so-called quantization region) defined by means of the centroid $C(kT) \in \mathbb{R}^n$ and the range vector $L(kT) \in \mathbb{R}^n$. The centroid $C(kT)$ is defined iteratively. First, let

$$\bar{x}(0^-) := C(0) := \underline{0} \in \mathbb{R}^n \quad (7)$$

and

$$L_i(0) \geq 2|(\Phi x(0))_i|, \quad i = 1, 2, \dots, n, \quad (8)$$

and define $\hat{x}(0)$ as the vector for which

$$\Phi(x(0) - \hat{x}(0)) \in \mathcal{R}_0^{SL(0)}.$$

The existence of $\hat{x}(0)$ fulfilling the condition above is guaranteed by (8) and Lemma 2. Solve the differential equation

$$\begin{aligned} \dot{\bar{x}} &= A\bar{x} + Bu \\ \bar{x}(0) &= \hat{x}(0) \end{aligned} \quad (9)$$

over the interval $[0, T)$ and set $C(T) := \bar{x}(T^-)$. For any $k \geq 1$, let $C(kT) := \bar{x}(kT^-)$, where $\bar{x}(kT^-)$ is obtained by

computing the solution of

$$\begin{aligned} \dot{\bar{x}} &= A\bar{x} + Bu \\ \bar{x}(kT) &= \hat{x}(kT) \end{aligned} \quad (10)$$

over the interval $[kT, (k+1)T)$, having defined $\hat{x}(kT)$ the vector for which

$$H^k \Phi(x(kT) - \hat{x}(kT)) \in \mathcal{R}_0^{SL(kT)},$$

where $L(\cdot)$ is the vector solution of the difference equation:

$$L((k+1)T) = \bar{F}HSL(kT) \quad (11)$$

with initial condition given in (8). The existence of $\hat{x}(kT)$ fulfilling the condition above is guaranteed by (8), (11) and Lemma 3 below.

Remark. The packet of bits, denoted by $s(kT)$, sent through the channel at time kT is the binary representation of the subregion in

$$\mathcal{R}_{H^k \Phi \bar{x}(kT^-)}^{L(kT)}$$

to which $H^k \Phi x(kT)$ belongs. As the number of these subregions is 2^R (see remark after Lemma 2), R bits would be enough to encode the information. However, for reasons which will be clear in the next section, we will need to take into account an additional subregion, namely the *overflow* region

$$\mathbb{R}^n \setminus \mathcal{R}_{H^k \Phi \bar{x}(kT^-)}^{L(kT)}.$$

The symbol s used to denote this region will be conventionally chosen equal to a sequence of 0 bits. Therefore, if a symbol $s(kT) = 0$ is generated at time kT , then we know that

$$H^k \Phi x(kT) \in \mathbb{R}^n \setminus \mathcal{R}_{H^k \Phi \bar{x}(kT^-)}^{L(kT)}.$$

Note finally that $B := \lceil \log_2(2^R + 1) \rceil$ bits will actually be used to encode information. \triangleleft

The following lemma shows that the equations of the encoder above are well-defined.

Lemma 3: Consider system (1) with $m(\cdot) = 0$ and let Assumptions 1 and 2 hold. Vector $\bar{x}(\cdot)$ generated by encoder (10), (11) with initial conditions (7), (8) satisfies:

$$|(H^k \Phi(x(kT) - \bar{x}(kT^-)))_i| \leq L_i(kT)/2 \quad (12)$$

for all integers $k \geq 0$ and any $i = 1, 2, \dots, n$.

Proof: The proof is by induction. For $k = 0$, by (8), the thesis trivially holds. Assume now that, for some $k \geq 0$, (12) is true. Then,

$$H^k \Phi(x(kT) - \bar{x}(kT^-)) \in \mathcal{R}_0^{L(kT)}.$$

By Lemma 2, we obtain that

$$H^k \Phi(x(kT) - \hat{x}(kT)) \in \mathcal{R}_0^{SL(kT)}.$$

Consider now the evolution of the quantity $x(\cdot) - \bar{x}(\cdot)$ over the interval $[kT, (k+1)T)$. It satisfies

$$\dot{x}(t) - \dot{\bar{x}}(t) = A(x(t) - \bar{x}(t)).$$

Hence, for $t \in [kT, (k+1)T)$, we have:

$$\begin{aligned} x(t) - \bar{x}(t) &= e^{A(t-kT)}(x(kT) - \bar{x}(kT)) \\ &= e^{A(t-kT)}(x(kT) - \hat{x}(kT)) \\ &= \Phi^{-1}\bar{F}(t-kT)\Phi(x(kT) - \hat{x}(kT)), \end{aligned}$$

where $\bar{F}(t-kT)$ denotes the matrix \bar{F} in which T is replaced by $t-kT$. By Lemma 1 The latter equality implies

$$\Phi(x(t) - \bar{x}(t)) = H^{-(k+1)}\bar{F}(t-kT)H^{k+1}\Phi(x(kT) - \hat{x}(kT))$$

or, equivalently,

$$H^{k+1}\Phi(x(t) - \bar{x}(t)) = \bar{F}(t-kT)H H^k\Phi(x(kT) - \hat{x}(kT)).$$

To proceed, set

$$h_i = \begin{cases} 0 & \text{if } i = 1 \\ h_{i-1} + n_{i-1} & \text{if } 1 < i \leq q. \end{cases} \quad (13)$$

For each $i = 1, 2, \dots, q$, for each $j = 1, 2, \dots, n_i$, from the previous equality we have:

$$\begin{aligned} (H^{k+1}\Phi(x(t) - \bar{x}(t)))_{h_i+j} &= \\ (\bar{F}(t-kT)H)_{h_i+j} H^k\Phi(x(kT) - \hat{x}(kT)) &= \\ \sum_{\ell=0}^{n_i-j} (\bar{F}(t-kT)H)_{h_i+j, h_i+j+\ell} (H^k\Phi(x(kT) - \hat{x}(kT)))_{h_i+j+\ell}. \end{aligned}$$

Letting $t \rightarrow (k+1)T^-$, the latter equality implies

$$\begin{aligned} |(H^{k+1}\Phi(x((k+1)T) - \bar{x}((k+1)T^-)))_{h_i+j}| &= \\ | \sum_{\ell=0}^{n_i-j} (\bar{F}H)_{h_i+j, h_i+j+\ell} (H^k\Phi(x(kT) - \hat{x}(kT)))_{h_i+j+\ell} | &\leq \\ \sum_{\ell=0}^{n_i-j} (\bar{F}H)_{h_i+j, h_i+j+\ell} |(H^k\Phi(x(kT) - \hat{x}(kT)))_{h_i+j+\ell}|. \end{aligned}$$

By the inductive hypothesis and the latter inequality,

$$\begin{aligned} |(H^{k+1}T(x((k+1)T) - \bar{x}((k+1)T^-)))_{h_i+j}| &\leq \\ \sum_{\ell=0}^{n_i-j} (\bar{F}H)_{h_i+j, h_i+j+\ell} S_{h_i+j+\ell} L(kT)/2 &= \\ \sum_{\ell=0}^{n_i-j} (\bar{F}H)_{h_i+j, h_i+j+\ell} S_{h_i+j+\ell, h_i+j+\ell} L_{h_i+j+\ell}(kT)/2 &= \\ \sum_{\ell=0}^{n_i-j} \sum_{p=1}^n (\bar{F}H)_{h_i+j, p} S_{p, h_i+j+\ell} L_{h_i+j+\ell}(kT)/2 &= \\ \sum_{\ell=0}^{n_i-j} (\bar{F}HS)_{h_i+j, h_i+j+\ell} L_{h_i+j+\ell}(kT)/2 &= \\ L_{h_i+j}((k+1)T)/2, \end{aligned} \quad (14)$$

where the latter equality holds by (11). This ends the proof. ■

Remark. Formula (12) can be written as

$$H^k\Phi(x(kT) - \bar{x}(kT^-)) \in \mathcal{R}_0^{L(kT)}.$$

By Lemma 2, there exists a vector $\hat{x}(kT)$ such that

$$H^k\Phi(x(kT) - \hat{x}(kT)) \in \mathcal{R}_0^{SL(kT)}. \quad (15)$$

Furthermore, by (11), there exist real numbers $0 < \bar{\mu}$ and $0 < \bar{\lambda} < 1$, for which

$$|L(kT)| \leq \bar{\mu} \bar{\lambda}^k |L(0)|. \quad (16)$$

Using the arguments of Lemma 3, by (15) and (16), it is straightforward to show that, for all $t \geq 0$,

$$|x(t) - \bar{x}(t)| \leq \mu(t) e^{-\lambda t} |L(0)|, \quad (17)$$

where $\mu(t)$ is any bounded signal satisfying, for $t \in [kT, (k+1)T)$ and $k \geq 0$,

$$\mu(t) \geq \|\Phi^{-1}\| \max_{\theta \in [0, T]} \|\bar{F}(\theta)\| \|H^{-k}\| \|S\| \bar{\mu} \bar{\lambda}^{-1},$$

and $\lambda = -|\ln \bar{\lambda}|/T$. Formula (17) points out that the encoded information can serve to estimation and control. In fact, any device deploying a decoder able to reproduce \bar{x} starting from the stream of symbols s is able to asymptotically estimate x . This estimate can also be used to devise a control action relying on the certainty equivalence principle. ◁

IV. FAULT DETECTION

In the previous section we have examined the functioning of the encoder. We have pointed out in the remark preceding Lemma 3 that the outcome of the encoding procedure is a stream of packets of B bits which are sent through the channel. The issue we address in this section is how this stream of encoded information can be used to a specific purpose, namely the purpose of detecting faults.

As already pointed out in [14], because of the encoding, there are faults which can not be detected despite of the observability property enjoyed by the process under monitoring. This suggests to cast the problem of detecting faults belonging to a suitable class. This was done in [14], from which we borrow the following definition:

Definition. Consider system (1) and encoder (10), (11). The fault detection problem with encoded full-state information is said to be solvable with respect to a class \mathcal{M} of faults if there exists a law $\varphi(\cdot)$ such that the signal

$$r(\cdot) = \varphi(s|_{[0, \cdot]}),$$

with $s|_{[0, \cdot]}$ the sequence of packets received through the channel, satisfies the properties:

- (i) $r(\cdot) = 0$ if $m(\cdot) = 0$;
- (ii) $r(\cdot) \neq 0$ if $m(\cdot) \in \mathcal{M}$ and $m(\cdot) \neq 0$. ◁

The following proposition characterizes a class of faults for which the detection problem is solvable.

Proposition 1: Let Assumptions 1 and 2 hold. The law

$$r(k) = \varphi(s(k)) = \begin{cases} 0 & \text{if } s(k) \neq 0 \\ 1 & \text{if } s(k) = 0 \end{cases}$$

solves the fault detection problem with encoded full-state information with respect to the class \mathcal{M} comprised by faults $m(\cdot)$ for which there exists an index $\tilde{k} \in \mathbb{Z}_+$ and a time $\tilde{t} \in [\tilde{k}T, (\tilde{k}+1)T)$ such that

$$\left| \int_{\tilde{t}}^{(\tilde{k}+1)T} (H^{\tilde{k}+1} \Phi e^{A((\tilde{k}+1)T-\tau)})_{h_i+j} M m(\tau) d\tau \right| > L_{h_i+j}((\tilde{k}+1)T) \quad (18)$$

for some $i = 1, 2, \dots, q$ and some $j = 1, 2, \dots, n_i$.

Proof: We start showing that property (i) is fulfilled. In fact, when $m(\cdot) = 0$ the conclusion of Lemma 3 holds and in particular it is possible to conclude that (see also the remark preceding the lemma)

$$H^k \Phi x(kT) \in \mathcal{R}_{H^k \Phi \bar{x}(kT^-)}^{L(kT)}.$$

This implies that for all $k \geq 0$, the symbol $s(kT)$ is different from 0 and therefore $r(k) = 0$ for all $k \geq 0$.

Consider now the case when $m(\cdot) \neq 0$. In particular, let \tilde{t} the first time for which $m(\tilde{t}) \neq 0$ and assume that \tilde{k} is the integer for which $\tilde{t} \in [\tilde{k}T, (\tilde{k}+1)T)$. Keeping in mind the proof of Lemma 3, we have

$$H^{\tilde{k}+1} \Phi(x(t) - \bar{x}(t)) = \bar{F}(t - \tilde{k}T) H^{\tilde{k}} \Phi(x(\tilde{k}T) - \hat{x}(\tilde{k}T)) + \int_{\tilde{t}}^t H^{\tilde{k}+1} \Phi e^{A(t-\tau)} M m(\tau) d\tau.$$

Letting $t \rightarrow (k+1)T^-$ and recalling (15), the latter equality implies

$$\begin{aligned} & |(H^{\tilde{k}+1} \Phi(x((\tilde{k}+1)T) - \bar{x}((\tilde{k}+1)T^-)))_{h_i+j}| \geq \\ & \left| \int_{\tilde{t}}^{(\tilde{k}+1)T} (H^{\tilde{k}+1} \Phi e^{A((\tilde{k}+1)T-\tau)})_{h_i+j} M m(\tau) d\tau \right| - \\ & L_{h_i+j}((\tilde{k}+1)T)/2, \end{aligned}$$

for each $i = 1, 2, \dots, q$ and each $j = 1, 2, \dots, n_i$. If (18) holds with $\tilde{k} = \bar{k}$ and $\tilde{t} = \bar{t}$, then the latter inequality yields

$$\begin{aligned} & |(H^{\tilde{k}+1} \Phi(x((\tilde{k}+1)T) - \bar{x}((\tilde{k}+1)T^-)))_{h_i+j}| > \\ & L_{h_i+j}((\tilde{k}+1)T)/2, \end{aligned} \quad (19)$$

which shows that fault $m(\cdot)$ causes the occurrence of overflow at time $(\tilde{k}+1)T$. Hence, $s((\tilde{k}+1)T) = 0$ and in turn $r((\tilde{k}+1)T) = 1$. On the other hand, if (18) does not hold with $\tilde{k} = \bar{k}$ and $\tilde{t} = \bar{t}$, (19) can still be true or it can not. In the former case the fault is detected, in the latter case we have necessarily

$$\begin{aligned} & |(H^{\tilde{k}+1} \Phi(x((\tilde{k}+1)T) - \bar{x}((\tilde{k}+1)T^-)))_{h_i+j}| \leq \\ & L_{h_i+j}((\tilde{k}+1)T)/2, \end{aligned}$$

for each $i = 1, 2, \dots, q$ and each $j = 1, 2, \dots, n_i$, that is

$$H^{\tilde{k}+1} \Phi(x((\tilde{k}+1)T) - \bar{x}((\tilde{k}+1)T^-)) \in \mathcal{R}_{H^{\tilde{k}+1}T(\bar{x}((\tilde{k}+1)T^-))}^{L((\tilde{k}+1)T)}.$$

As a consequence, by Lemma 2, there exists a vector $\hat{x}((\tilde{k}+1)T)$ such that

$$H^{\tilde{k}+1} \Phi(x((\tilde{k}+1)T) - \hat{x}((\tilde{k}+1)T)) \in \mathcal{R}_0^{SL((\tilde{k}+1)T)}. \quad (20)$$

By applying the same arguments as above, one proves that

$$\begin{aligned} & |(H^{\tilde{k}+2} \Phi(x((\tilde{k}+2)T) - \bar{x}((\tilde{k}+2)T^-)))_{h_i+j}| \geq \\ & \left| \int_{(\tilde{k}+1)T}^{(\tilde{k}+2)T} (H^{\tilde{k}+2} \Phi e^{A((\tilde{k}+2)T-\tau)})_{h_i+j} M m(\tau) d\tau \right| - \\ & L_{h_i+j}((\tilde{k}+2)T)/2, \end{aligned}$$

for each $i = 1, 2, \dots, q$ and each $j = 1, 2, \dots, n_i$. If (18) holds with $\tilde{k} = \bar{k} + 1$ and $\tilde{t} = (\bar{k} + 1)T$, then the fault is surely detected at time $(\bar{k} + 2)T$. If not, then it can still happen that either an overflow symbol is generated, in which case occurrence of the fault is inferred, or $H^k \Phi(x((\bar{k} + 2)T))$ belongs to the quantization region at time $(\bar{k} + 2)T$ and therefore no fault can be detected at this time. Hence, the evolution of the system must be studied over the next time interval. Iteration of these arguments yields the proof. ■

In the result above, considering a class of faults with a “sufficiently large” magnitude at some time allows a prompt detection of the fault based on the fact that the fault drives away the state from the quantization region, an event was guaranteed to never happen in the un-faulty situation. The main advantage yielded by such a class of faults is a reduction in the complexity of the monitoring unit, which in this case becomes a simple decision logic.

Example. Consider a linear system of the form (1) with

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = M = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

In this case, $H = \Phi = I_2$ and condition (18) can be rewritten as:

$$\left| \int_{\tilde{t}}^{(\tilde{k}+1)T} ((\tilde{k}+1)T - \tau) m(\tau) d\tau \right| > L_1((\tilde{k}+1)T)$$

for $h_i = 0$ and $j = 1$ and

$$\left| \int_{\tilde{t}}^{(\tilde{k}+1)T} m(\tau) d\tau \right| > L_2((\tilde{k}+1)T)$$

for $h_i = 0$ and $j = 2$, where

$$L_1(kT) = \frac{1}{2k} L_1(0) + \frac{T}{2} \frac{1}{2^{k-1}} k L_2(0), \quad L_2(kT) = \frac{1}{2k} L_2(0).$$

Take for the sake of simplicity $m(\tilde{t}) = \bar{m} = \text{const.}$ As there always exists an index k for which

$$|\bar{m}|T > \frac{1}{2^{k+1}} L_2(0)$$

the second condition above shows that the fault will be ultimately detected. ◁

V. CONCLUSION

We have proposed a method to detect faults from encoded information. We operated under the assumption that the information has been encoded by time-varying encoders and that it can serve not only to fault detection but also for estimation and control. In other words, the class of time-varying encoders used to encode information is not specific to the detection problem under consideration. The method relies on the observation that faults cause the encoders to generate a stream of bits sensibly different from the stream generated in the un-faulty case. In particular, the class of faults considered in this paper causes the encoder to generate a stream of bits from which the occurrence of the fault is inferred by a simple decision logic. For classes of faults which are "less evident", monitoring unit may implement a more complex structure than a decision logic. Many extensions are possible, among which: fault detection and isolation of multiple faults and fault detection for nonlinear processes.

VI. REFERENCES

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